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EQUATIONS OF ELASTOPLASTIC DEFORMATION FOR ARBITRARY VALUES OF THE ROTATIONS AND DEFORMATIONS

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In many solids, for example, in metallic bodies, for arbitrary values of the rotations and deformations of the elements of the body, the components of the deviator of the elastic deformations are quantities on the order of the ratio of the shear strength to the Young modulus and, consequently, are small in comparison with unity. Below, on the basis of the results of [1], equations are formulated for the isotropic elastic and ideal elastoplastic deformation of such bodies. A comparison is made between the equations obtained and known equations [2-4]. For simplicity in writing the equations, only adiabatic deformations are discussed below.

1. Equations of Elastic Deformation in the Case of Small Components of the Deviator of the Deformations

We denote by $\mathbf{Э}_\alpha$, $\mathbf{Э}^\beta$ $(\alpha, \beta = 1, 2, 3)$ the basis vectors of a Lagrangian system of coordinates, generated by the Cartesian system of coordinates x^i with the basis vectors $\mathbf{k}_i = \mathbf{k}^i$ $(i = 1, 2, 3)$.

Let $\hat{\gamma}_{\alpha\beta}\mathbf{Э}^\alpha\mathbf{Э}^\beta = \hat{\gamma}^{\sigma\lambda}\mathbf{Э}_\sigma\mathbf{Э}_\lambda = \gamma_{ij}\mathbf{k}^i\mathbf{k}^j$ be some symmetrical tensor. Differentiating the formulas for the connection between the components $\hat{\gamma}_{\alpha\beta}$, $\hat{\gamma}^{\sigma\lambda}$ and the components γ_{ij}, we find

$$(d\hat{\gamma}_{\alpha\beta}/dt)\mathbf{Э}^\alpha\mathbf{Э}^\beta = (D\gamma_{ij}/Dt + \gamma_{sj}e_{si} + \gamma_{si}e_{sj})\mathbf{k}^i\mathbf{k}^j,$$
$$(d\hat{\gamma}^{\alpha\beta}/dt)\mathbf{Э}_\alpha\mathbf{Э}_\beta = (D\gamma_{ij}/Dt - \gamma_{sj}e_{si} - \gamma_{si}e_{sj})\mathbf{k}^i\mathbf{k}^j, \tag{1.1}$$

where $e_{ij} = (1/2)(\partial u_i/\partial x^j + \partial u_j/\partial x^i)$; u_i are the components of the velocity vector; $D\gamma_{ij}/Dt$ is a Jaumann derivative [5]

$$D\gamma_{ij}/Dt = d\gamma_{ij}/dt + \gamma_{ki}\omega_{kj} + \gamma_{kj}\omega_{ki},$$
$$\omega_{ij} = (1/2)(\partial u_i/\partial x^j - \partial u_j/\partial x^i).$$

From (1.1), specifically, it follows that

$$D\varepsilon_{ij}/Dt + \varepsilon_{ki}e_{kj} + \varepsilon_{jk}e_{ki} = e_{ij}, \tag{1.2}$$

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where

$$\varepsilon_{ij}\mathbf{k}^i\mathbf{k}^j = \hat{\varepsilon}_{\alpha\beta}\partial^\alpha\partial^\beta; \quad \hat{\varepsilon}_{\alpha\beta} = (1/2)(\hat{g}_{\alpha\beta} - \delta_{\alpha\beta}); \quad \hat{g}_{\alpha\beta} = \partial_\alpha\cdot\partial_\beta.$$

Obviously, Eqs. (1.2) can be written in the form

$$D\varepsilon_{ij}/Dt + (2/3)\varepsilon e_{ij} = a_{ij}, \quad \varepsilon = \varepsilon_{ij}\delta_{ij}, \\ a_{ij} = e_{ij} - \varepsilon'_{ik}e_{kj} - \varepsilon'_{jk}e_{ki}, \ \varepsilon'_{ij} = \varepsilon_{ij} - \frac{1}{3}\delta_{ij}\varepsilon. \tag{1.3}$$

Since, in the system of coordinates x^i, formed by the principal axes of the deformations,

$$a_{11} = (1 - 2\varepsilon'_{11})\, e_{11}, \ a_{12} = (1 + \varepsilon'_{33})\, e_{12}, \ \ldots,$$

then for

$$1 + \varepsilon'_{ij} \approx 1 \tag{1.4}$$

the tensor with the components a_{ij} is equivalent to the tensor of the deformation rates e_{ij}. Therefore, in the case (1.4), instead of (1.3) we can use the equation

$$D\varepsilon_{ij}/Dt = (1-(2/3)\varepsilon)e_{ij}. \tag{1.5}$$

From (1.5) and the equation of continuity

$$d\rho/dt + \rho e = 0$$

it follows that

$$D\varepsilon'_{ij}/Dt = \left(1 - \frac{2}{3}\varepsilon\right)e'_{ij}, \ d\varepsilon/dt = \left(1 - \frac{2}{3}\varepsilon\right)e, \\ e'_{ij} = e_{ij} - \frac{1}{3}\delta_{ij}e, \ e = \delta_{ij}e_{ij}, \ \rho = \rho_0\left(1 - \frac{2}{3}\varepsilon\right)^{3/2}, \tag{1.6}$$

where ρ_0 is the density in the undeformed state.

Analogously, we find from (1.1) and (1.6) that, in the case (1.4), to write Eqs. (1.6) in a Lagrangian system of coordinates the formulas

$$d\hat{\varepsilon}'_{\alpha\beta}/dt\, \partial^\alpha\partial^\beta = d\hat{\varepsilon}'^{\gamma\lambda}/dt\, \partial_\gamma\partial_\lambda = D\varepsilon'_{ij}/Dt k^i k^j, \\ \hat{\varepsilon}'_{\alpha\beta} = \hat{\varepsilon}_{\alpha\beta} - \frac{1}{3}\hat{g}_{\alpha\beta}\varepsilon, \ \hat{\varepsilon}'^{\gamma\lambda} = \hat{g}^{\gamma s}\hat{g}^{\lambda k}\hat{\varepsilon}'_{sk}, \ \hat{g}^{\gamma s} = \partial^\gamma\cdot\partial^s \tag{1.7}$$

must be used.

We assume that, for isotropic elastic deformation, the internal energy E is a function of the entropy S and the invariants ε, Γ_2, and Γ_3 of the tensor of the deformations

$$E = E(\varepsilon, \Gamma_2, \Gamma_3, S), \ \Gamma_2 = \frac{1}{2}\varepsilon'_{ij}\varepsilon'_{ij}, \ \Gamma_3 = \frac{1}{3}\varepsilon'_{ik}\varepsilon'_{kj}\varepsilon'_{ij}. \tag{1.8}$$

From (1.6) and (1.8) we find

$$dE/dt = \left(1 - \frac{2}{3}\varepsilon\right)\left[e\partial E/\partial\varepsilon + \varepsilon'_{ij}\partial E/\partial\Gamma_2 + \varepsilon'_{ik}\varepsilon'_{kj}\partial E/\partial\Gamma_3\right]e'_{ij} + TdS/dt, \ T = \partial E/\partial S. \tag{1.9}$$

Since elastic deformation is a reversible process, then from the law of conservation of energy

$$\sigma_{ij}e_{ij} = \rho dE/dt \tag{1.10}$$

and (1.6) and (1.9) there follow the equations

$$\sigma = \alpha\,\partial E/\partial\varepsilon, \ \sigma'_{ij} = \beta\varepsilon'_{ij} + \gamma\left(\varepsilon'_{ik}\varepsilon'_{kj} - \frac{2}{3}\Gamma_2\delta_{ij}\right), \ dS/dt = 0, \\ \alpha = \rho_0\left(1 - \frac{2}{3}\varepsilon\right)^{5/2}, \ \beta = \alpha\,\partial E/\partial\Gamma_2, \ \gamma = \alpha\,\partial E/\partial\Gamma_3. \tag{1.11}$$

For elastic deformation, the correspondence between the values of the stresses and deformations must be mutually single-valued. Therefore, in order for the equations (1.11) to be the equations of elastic deformation, it is required that they uniquely determine the values of the deformations from given values of the stresses. From (1.11) we find

$$\sigma'_{ik}\sigma'_{kj} = A\,\varepsilon'_{ij} + B\,\varepsilon'_{ik}\varepsilon'_{kj} + C\,\delta_{ij}, \\ A = \gamma\left(\frac{2}{3}\beta\Gamma_2 + \gamma\Gamma_3\right), \ B = \beta^2 - \frac{1}{3}\gamma^2\Gamma_2, \ C = 2\gamma\left(\beta\Gamma_3 + \frac{2}{9}\Gamma_2^2\gamma\right),$$

$$J_2 = \frac{1}{2}\sigma'_{ij}\sigma'_{ij} = \beta^2\Gamma_2 + \frac{1}{3}\gamma^2\Gamma_2^2 + 3\beta\gamma\Gamma_3,$$

$$J_3 = \frac{1}{3}\sigma'_{ik}\sigma'_{kj}\sigma'_{ij} = \frac{2}{3}\gamma\Gamma_2\left(\beta^2\Gamma_2 - \frac{1}{9}\gamma^2\Gamma_2^2 + \frac{3}{2}\gamma\beta\Gamma_3\right) + (\beta^3 + \gamma^3\Gamma_3)\,\Gamma_3.$$

From this, and from (1.11), it follows that in the case

$$\partial(\alpha\partial E/\partial\varepsilon)/\partial\varepsilon \neq 0,\quad \beta^3 - \gamma^2(\beta\Gamma_2 + \gamma\Gamma_3) \neq 0,$$

$$(\partial J_2/\partial\Gamma_2)(\partial J_3/\partial\Gamma_3) - (\partial J_2/\partial\Gamma_3)(\partial J_3/\partial\Gamma_2) \neq 0$$

Eqs. (1.11) are solvable for ε and ε'_{ij}, and they can be written in the form

$$\varepsilon'_{ij} = a\,\sigma'_{ij} + bQ'_{ij},\quad Q'_{ij} = \sigma'_{ik}\sigma'_{kj} - \frac{2}{3}J_2\delta_{ij},\qquad \sigma = \sigma(\varepsilon,\ J_2,\ J_3,\ S), \tag{1.12}$$

where a and b can be regarded as functions of ε, J_2, J_3, and S.

For Eqs. (1.6) and (1.12) with given functions a, b, and σ to be the equations of elastic deformation, it is sufficient that a, b, and σ satisfy the conditions for the solvability of Eqs. (1.12) for σ'_{ij} and ε

$$\begin{gathered}\partial\sigma/\partial\varepsilon \neq 0,\quad a^3 - b^2(aJ_2 + bJ_3) \neq 0,\\ (\partial\Gamma_2/\partial J_2)(\partial\Gamma_3/\partial J_3) - (\partial\Gamma_2/\partial J_3)(\partial\Gamma_3/\partial J_2) \neq 0,\\ \Gamma_2 = a^2J_2 + \frac{1}{3}b^2J_2^2 + 3\,abJ_3,\\ \Gamma_3 = \frac{2}{3}bJ_2\left(a^2J_2 - \frac{1}{9}b^2J_2^2 + \frac{3}{2}abJ_3\right) - J_3(a^3 + b^3J_3),\end{gathered} \tag{1.13}$$

and the law of conservation of energy (1.10) for the energy E regarded as a function of ε, J_2, J_3, and S. Since in accordance with (1.6) and (1.12) it holds that

$$\begin{gathered}\left(1 - \frac{2}{3}\varepsilon\right)e'_{ij} = D\left(a\,\sigma'_{ij} - bQ'_{ij}\right)/Dt,\\ \left(1 - \frac{2}{3}\varepsilon\right)\sigma'_{ij}e'_{ij} = adJ_2/dt + 2\,bdJ_3/dt + 2\,J_2da/dt + 3\,J_3db/dt.\end{gathered} \tag{1.14}$$

then the law of conservation of energy (1.10) for $E = E(\varepsilon, J_2, J_3, S)$ will be satisfied if

$$\begin{gathered}\alpha\partial E/\partial J_2 = a + 2J_2\partial a/\partial J_2 + 3J_3\partial b/\partial J_2,\\ \alpha\partial E/\partial J_3 = 2(b + J_2\partial a/\partial J_3) + 3J_3\partial b/\partial J_3,\\ \alpha\partial E/\partial\varepsilon = \sigma + 2J_2\partial a/\partial\varepsilon + 3J_3\partial b/\partial\varepsilon,\end{gathered}$$

and, consequently, a, b, and σ must satisfy the conditions

$$\begin{gathered}\partial a/\partial J_3 = \partial b/\partial J_2,\quad a + 2J_2\partial a/\partial J_2 + 3J_3\partial b/\partial J_2 = (3/5)(1 - (2/3)\varepsilon)\,(\partial\sigma/\partial J_2 + \partial a/\partial\varepsilon),\\ 2(b + J_2\partial a/\partial J_3) + 3J_3\partial b/\partial J_3 = (3/5)(1 - (2/3)\varepsilon)(\partial\sigma/\partial J_3 + \partial b/\partial\varepsilon).\end{gathered} \tag{1.15}$$

Thus, the equations of elastic deformation in the case (1.4) can be constructed, giving a, b, and σ as functions of ε, J_2, J_3, and S in such a way that the conditions (1.13) and (1.15) will be satisfied. Specifically, these conditions will be satisfied for

$$\begin{gathered}a = 1/2\mu\alpha,\quad \mu = \mu(\varepsilon,\ S),\quad b = 0,\quad \sigma = \alpha\partial\psi/\partial\varepsilon - J_2d\,(1/2\mu)/\alpha d\varepsilon,\\ \psi = \psi(\varepsilon,\ S),\quad E = J_2/2\mu\alpha^2 + \psi(\varepsilon,\ S).\end{gathered} \tag{1.16}$$

If the relative change in the density is small in comparison with unity, then the value of $1 - (2/3)\varepsilon$ in (1.5) and (1.6) and in all following equations can be replaced by unity.

If the angular velocities of the elements of the medium and the shear deformations are quantities on the same order of magnitude,

$$\omega_{ij} \sim e_{ij} \sim \partial u_i/\partial x^j\quad (i \neq j),$$

then the tensor with the components

$$e_{ij} - \varepsilon'_{ik}\partial u_k/\partial x^j - \varepsilon'_{jk}\partial u_k/\partial x^i$$

is equivalent to the tensor of the deformation rates. In this case, the Jaumann derivative in (1.5)-(1.7), (1.14) can be replaced by the derivative with respect to the time.

2. Equations of Elastoplastic Deformation for Small Components of the Deviator of the Elastic Deformations

For elastoplastic deformation, along with reversible (elastic) deformations, there are irreversible (plastic) deformations. We denote

$$e_{ij} = e^e_{ij} - e^p_{ij}, \quad \varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^p_{ij}, \tag{2.1}$$

where e^e_{ij} and e^p_{ij} are the rates of the elastic and plastic deformations; ε^e_{ij} and ε^p_{ij} are the elastic and plastic deformations. From (1.1), (1.2), and (2.1) it follows that

$$D\varepsilon^e_{ij}/Dt - \varepsilon^e_{is}e_{sj} + \varepsilon^e_{js}e_{si} = e^e_{ij} = e_{ij} - e^p_{ij}. \tag{2.2}$$

If the components of the deviator of the elastic deformations are small in comparison with unity,

$$1 + \varepsilon'^e_{ij} \approx 1, \quad \varepsilon'^e_{ij} = \varepsilon^e_{ij} - \frac{1}{3}\delta_{ij}\delta_{ks}\varepsilon^e_{ks}, \tag{2.3}$$

then the tensor with the components α_{ij},

$$\alpha_{ij} = e_{ij} - \varepsilon'^e_{is}e_{sj} - \varepsilon'^e_{js}e_{si}$$

is equivalent to the tensor of the deformation rates e_{ij}, and Eqs. (2.2) can be written in the form

$$D\varepsilon^e_{ij}/Dt + \frac{2}{3}e_{ij}\delta_{ks}\varepsilon^e_{ks} = e^e_{ij} = e_{ij} - e^p_{ij}. \tag{2.4}$$

We assume that the volume of an element of the medium varies elastically:

$$e = \delta_{ij}e_{ij} = \delta_{ks}e^e_{ks}. \tag{2.5}$$

From (2.4), (2.5), and the equation of continuity we find

$$\left(1 - \frac{2}{3}\varepsilon\right)e'_{ij} = D\varepsilon'^e_{ij}/Dt + e^p_{ij},$$
$$d\varepsilon/dt = \left(1 - \frac{2}{3}\varepsilon\right)e, \quad \rho = \rho_0\left(1 - \frac{2}{3}\varepsilon\right)^{3/2}. \tag{2.6}$$

If, as the dependence of ε'^e_{ij} on σ_{ij}, we use Eqs. (1.12) and (1.16), and, as the dependence of e^p_{ij} on σ_{ij}, we use the equations of ideal plastic flow with the Mises plasticity condition [6], we then obtain

$$\varepsilon'^e_{ij} = \sigma'_{ij}/2\mu\alpha, \quad e^p_{ij} = \lambda\sigma'_{ij},$$
$$\sigma = \alpha\partial\psi/\partial\varepsilon - J_2 d(1/2\mu)/\alpha d\varepsilon, \quad \mu = \mu(\varepsilon, S), \quad \alpha = \rho_0(1 - (2/3)\varepsilon)^{5/2},$$
$$\psi = \psi(\varepsilon, S), \tag{2.7}$$
$$E = \psi + J_2/2\mu\alpha^2, \quad \alpha T dS/dt = 2\tau^2\lambda, \quad T = \partial E/\partial S;$$
$$\lambda = 0, \text{ if } \quad f < 0 \text{ or } f = 0, \quad df/dt < 0;$$
$$\lambda \geqslant 0, \text{ if } \quad f = 0, \ df/dt = 0, \ f = J_2 - \tau^2, \ \tau = \tau(\varepsilon, S) \tag{2.8}$$

(τ is the shear strength).

For a medium satisfying condition (2.3), Eqs. (2.6)-(2.8) form a system of equations of elastoplastic deformation, correct with an arbitrary value of the rotations and deformations of the elements of the medium. Equations (2.6)-(2.8) coincide with the known equations [2-4] in the case where μ = constant, τ = const, and the relative change in the density is small in comparison with unity; consequently,

$$1 - (2/3)\varepsilon = (\rho/\rho_0)^{2/3} \approx 1, \quad \alpha \approx \rho_0, \quad d\varepsilon/dt \approx e.$$

In writing Eqs. (2.6)-(2.8) in a Lagrangian system of coordinates, the dependence of the components of the deviator of the deformation on the stresses can be written, using (1.6), in the form

$$\left(1 - \frac{2}{3}\varepsilon\right)\widehat{e}'_{\alpha\beta} = d\widehat{\varepsilon}'^e_{\alpha\beta}/dt + \lambda\widehat{\sigma}'_{\alpha\beta}, \quad \widehat{\varepsilon}'^e_{\alpha\beta} = \widehat{\sigma}'_{\alpha\beta}/2\mu\alpha.$$

The remaining equations (2.6)-(2.8) do not change.

Since in accordance with (2.6) and (2.7)

$$\left(1 - \frac{2}{3}\varepsilon\right)\sigma'_{ij}e'_{ij} = \frac{1}{2}dJ_2/\mu\alpha\, dt + J_2 d(1/\mu\alpha)/dt + 2J_2\lambda,$$

then the conditions (2.8), determining the function λ in (2.7), can be replaced by the conditions

$$\lambda = \frac{1}{2} c\,\omega/\tau^2, \quad \omega = \left(1 - \frac{2}{3}\varepsilon\right)\sigma'_{ij}e'_{ij} - \tau\, d\,(\tau/\mu\alpha)/dt,$$
$$c = 0, \quad \text{if} \quad f < 0 \quad \text{or} \quad f = 0, \quad \omega \leqslant 0;$$
$$c = 1, \quad \text{if} \quad f = 0, \quad \omega > 0.$$

For the computation of small increments of the stresses for a small interval of time from the deformation rates, instead of Eqs. (2.6)-(2.8), use can be made of a procedure, proposed in [4], for correction of the deviator of the stresses. Here the increments of the stresses before correction are calculated using Eqs. (1.11) and (1.16).

Using (1.12), (1.13), (1.15), and (2.6), for a medium with the condition (2.3), the equations of elastoplastic deformation can be formulated with a more general law of elastic deformation than in (2.6)-(2.8).

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NONISOTHERMAL NONLINEAR WAVES IN A ROD MADE OF A DISSIPATIVE RUBBERLIKE MATERIAL

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Article [1] discussed in the isothermal approximation, a wave propagating in an elastoviscous rod and gave a numerical solution to the problem of the impact of a rod of finite length on a rigid barrier. With the presence of strong geometrical and physical nonlinearities in the determining equations, waves of very great intensity can be propagated in the rods, where the effects of nonisothermicity are considerable with the propagation of the waves. The present article is devoted to an investigation of these questions.

1. Basic Equations

With the study of the motion of the rods, as in [1], we shall use a description averaged over the cross section. The material of the rod is assumed to be incompressible with the density ρ_0.

The equations of the mass balance, momentum, and energy in a "rod approximation" have the form

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x}(fv) = 0, \quad \frac{\partial}{\partial t}(fv) + \frac{\partial}{\partial x}\left(fv^2 - \rho_0^{-1} f\sigma\right) = 0,$$
$$\frac{\partial}{\partial t}\{\rho_0 f(U + v^2/2)\} + \frac{\partial}{\partial x}\{\rho_0 fv(U + v^2/2\} = \frac{\partial}{\partial x}(fv\sigma - q) + \alpha\sqrt{f}(T - T_0). \tag{1.1}$$

where f is the area of the transverse cross section of the rod; v is the mean velocity over the cross section; σ is the mean normal stress over the cross section (determined as in the homogeneous case, using the condition of the reversion of the stresses to zero at the free surface of the rod); U is the specific internal energy; q is the longitudinal heat flux; T is the mean temperature over the cross section of the rod; T_0 is the tempera-

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